**Unit 4 Day 10 - Key Features of Rational Graphs**

* Graphs of rational functions can either be continuous or discontinuous

$$y=\frac{x-1}{x^{2}+1}$$

* Continuous: have no \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ or breaks in the graph
	+ Discontinuous: a graph that has holes or \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
	+ Discontinuities can be:

$$y=\frac{1}{x^{2}-1}$$

* + 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Asymptotes
		2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Asymptotes
		3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Vertical Asymptotes**

* Vertical asymptotes are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_that the graph doesn’t pass through. (It gets really close but never crosses)
	+ - Vertical asymptote will be at x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
		- They occur where the denominator = \_\_\_\_\_\_\_\_\_\_\_\_\_
		- The find vertical asymptotes, set the denominator equal to 0 and \_\_\_\_\_\_\_\_\_\_\_(usually just change the sign)
1. Factor number and denominator and check for a common factor.
2. Set the numerator equal to 0.
3. Solve.

Find the vertical asymptote(s) of $y=\frac{3}{x+2}.$

1. Nothing to factor.
2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Vertical asymptote at \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Find the vertical asymptote(s) of $y=\frac{x+5}{x^{2}-9}.$

1. Factor number and denominator and check for a common factor.
2. Set the numerator equal to 0.
3. Solve.
4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_--> \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
6. Vertical asymptotes at \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Find the vertical asymptote(s) of $y=\frac{x^{2}-8x+7}{x+3}.$

1. Factor number and denominator and check for a common factor.
2. Set the numerator equal to 0.
3. Solve.
4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
6. Vertical asymptote(s) at\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Horizontal Asymptotes**

* If the degree in the denominator is higher than the degree in the numerator, there is a horizontal asymptote at \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$$y=\frac{x^{2}+3x+3}{x^{3}-8}$$

* If the degree in the numerator is higher than the degree in the denominator there is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$$y=\frac{x^{3}-8}{x^{2}+3x+3}$$

* If the degrees are the same, the there is a horizontal asymptote at $\left(\frac{coefficient of numerator}{coefficient of denominator}\right)$

$$y=\frac{2x^{2}-2}{3x^{2}-3x+6}$$

Find the horizontal asymptote(s):

1. $y=\frac{3x+5}{x-2}$
2. $y=\frac{x^{2}+2}{2x}$
3. $y=\frac{x+2}{2x^{2}-4}$

**Holes in the Graph:**

* Holes occur if the numerator and denominator have a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ factor
1. Factor numerator and denominator and check for common factor
2. Change the sign of the common factor
3. Plug x back into what’s left in the equation.

Find the hole of $y=\frac{x^{2}-3x+2}{x-2}$

Hole @ (\_\_\_\_\_\_, \_\_\_\_\_\_\_)

1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Common factor a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 hole at x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Find the hole of $y=\frac{x^{2}-4}{x+2}$

1. Factor numerator and denominator and check for common factor
2. Change the sign of the common factor
3. Plug x back into what’s left in the equation.

Hole @ (\_\_\_\_\_\_, \_\_\_\_\_\_\_)

1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Common factor a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 hole at x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Find the hole of $y=\frac{x^{2}+x-6}{x^{2}-x-2}$

1. Factor numerator and denominator and check for common factor
2. Change the sign of the common factor
3. Plug x back into what’s left in the equation.

Hole @ (\_\_\_\_\_\_, \_\_\_\_\_\_\_)

1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Common factor a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 hole at x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 🡪 \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. **Identify the key features of** $\frac{x^{2}-x}{-4x+12}$

**Factor = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

* + **Vertical Asymptote:**
	+ **Horizontal Asymptote:**
	+ **Hole(s):**
1. **Identify the key features of** $\frac{2x-6}{x^{2}-x-6}$

**Factor = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

* + **Vertical Asymptote:**
	+ **Horizontal Asymptote:**
	+ **Hole(s):**
1. **Identify the key features of** $\frac{x^{2}+7x+12}{2x^{2}+4x-6}$

**Factor = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

* + **Vertical Asymptote:**
	+ **Horizontal Asymptote:**
	+ **Hole(s):**
1. **Identify the key features of** $\frac{x^{2}-3x-4}{4x^{2}-16}$

**Factor = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

* + **Vertical Asymptote:**
	+ **Horizontal Asymptote:**
	+ **Hole(s):**