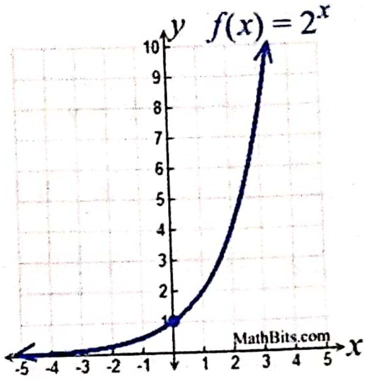
Unit 2 – Key Features of Exponential Graphs



- The basic form for an exponential equation is $y = a \cdot b^x$
  - a is the start value / y-intercept (unless the graph is translated)
  - b represents the growth/decay
    - If b is > 1 the equation represents growing
    - If b is < 1 the equation represents decay
- Exponential equations will always have a horizontal asymptote or a line the graph approaches but never crosses
  - To create an equation for a vertical line y = #

Transformations of Exponential Functions

- Just like the other functions we learned about, exponential functions can be moved up, down, left, or right or stretched or compressed either vertically or horizontally
- A value added/subtracted at the end of an exponential function will shift the graph either up or down
  - \+ moves the graph up
  - – moves the graph down
- A value added/subtracted with the exponent will move the graph either left or right
  - \+ moves the graph left
  - – moves the graph right
- A value multiplied outside parenthesis or away from the exponent will stretch or compress the graph Vertically
  - Numbers greater than 1 stretch the graph vertically
  - Numbers less than 1 compress the graph vertically
- A value multiplied with the exponent will stretch or compress the graph horizontally
  - Numbers greater than 1 compress the graph horizontally
  - Numbers less than 1 stretch the graph horizontally

For each example below, determine which way the function was moved from either parent function $y = 2^x$

1. $y = 2^{x-1}$

~~UP or DOWN~~ ______

LEFT or RIGHT 1

2. $y = 2^x - 1$

UP or DOWN 1

~~LEFT or RIGHT~~ ______

3. $y = 3^{x+4}$

~~UP or DOWN~~ ______

LEFT or RIGHT 4

4. $y = 2^{x+4}$

~~UP or DOWN~~ ______

LEFT or RIGHT 4

5. $y = 5^x - 6$

UP or DOWN 6

~~LEFT or RIGHT~~ ______

6. $y = 2^x + 4$

UP or DOWN 4

~~LEFT or RIGHT~~ ______

7. $y = 2^{x+3} - 5$

UP or DOWN 5

LEFT or RIGHT 3

8. $y = 3^{x-4} - 5$

UP or DOWN 5

LEFT or RIGHT 4

9. $y = 2^{x-2} + 6$

UP or DOWN 6

LEFT or RIGHT 2

For each example below, decide if the graph was effected vertically or horizontally by either a stretch or a compression from either parent function $y = 2^x$ or $y = log(x)$.

1. $y = 3 \cdot 2^x$

Vertical or Horizontal

Stretch or Compression

2. $y = 2^{3x}$

Vertical or Horizontal

Stretch or Compression

3. $y = 2^{\frac{1}{2}x}$

Vertical or Horizontal

Stretch or Compression

4. $y = 4(3^x)$

Vertical or Horizontal

Stretch or Compression

5. $y = \frac{1}{4} \cdot 2^x$

Vertical or Horizontal

Stretch or Compression

6. $y = 2^{\frac{1}{3}x}$

Vertical or Horizontal

Stretch or Compression

7. $y = 6^{(5x)}$

Vertical or Horizontal

Stretch or Compression

8. $y = 5 \cdot 2^x$

Vertical or Horizontal

Stretch or Compression

9. $y = 2^{4x}$

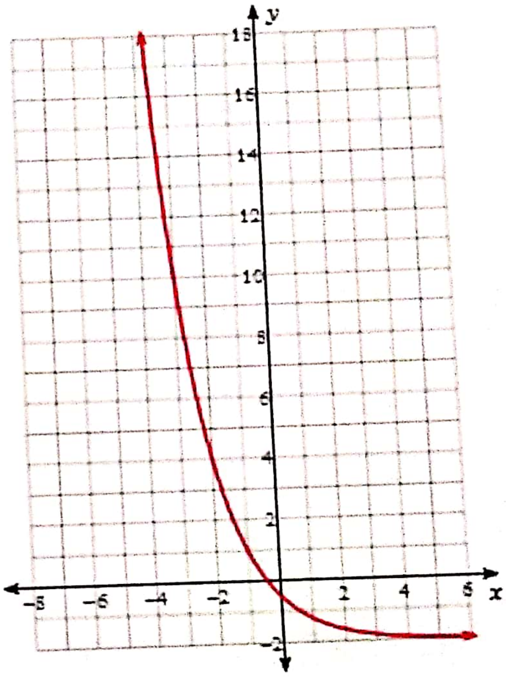
Vertical or Horizontal

Stretch or Compression

Key Features of Exponential Graphs

- In addition to the key features we talked about with other graphs (domain, range, increasing/decreasing) we will be talking about end behavior
  - End behavior will ask you to look at your y-values as x approaches ∞ (right side of the graph) and x approaches −∞ (left side of your graph)
- We will also identify the asymptote or the line our graph will get very close to but won't touch

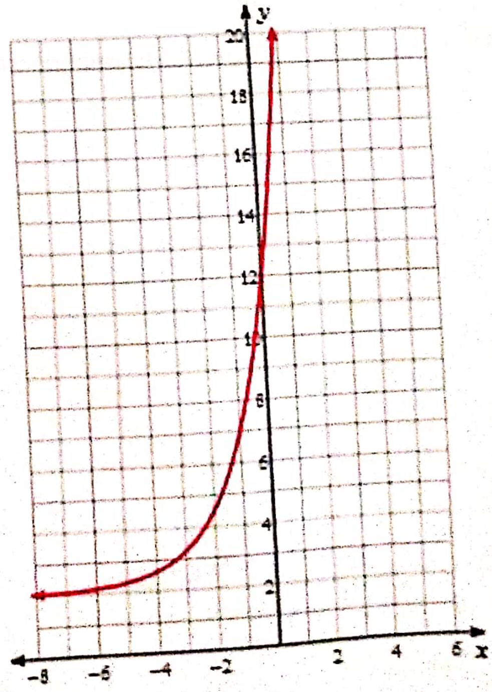
(#1) The graph of $y = 3\left(\frac{1}{2}\right)^{x+1} - 2$ is shown below. Use this graph to identify the following key features.



- Domain: $(-\infty, \infty)$
- Range: $(-2, \infty)$
- Increasing/Growth or Decreasing/Decay
- Asymptote at $y = -2$
- Y-intercept at -0.5
- End behavior:

$x \to \infty, y \to -2$ $\quad$ $x \to -\infty, y \to \infty$

(#2) The graph of $y = 3\left(\frac{1}{2}\right)^{x+1} + 2$ is shown below. Use this graph to identify the following key features.



- Domain: $(-\infty, \infty)$
- Range: $(2, \infty)$
- Increasing/Growth or Decreasing/Decay
- Asymptote at $y = 2$
- Y-intercept at 12
- End behavior:

$x \to \infty, y \to \infty$ $\quad$ $x \to -\infty, y \to 2$

- From the previous two examples, do you notice any relationship between the equation and the vertical asymptote? The vertical shift (up/down) was the same as the vertical asymptote
- For exponential equations, the vertical asymptote will always be at y = the value of the vertical shift and the range will be (#, ∞)

For each equation below, identify the vertical asymptote as well as the domain and range.

1. $y = 3(2)^x + 3$

   Vertical asymptote @ y = 3

   Domain: $(-\infty, \infty)$

   Range: $(3, \infty)$

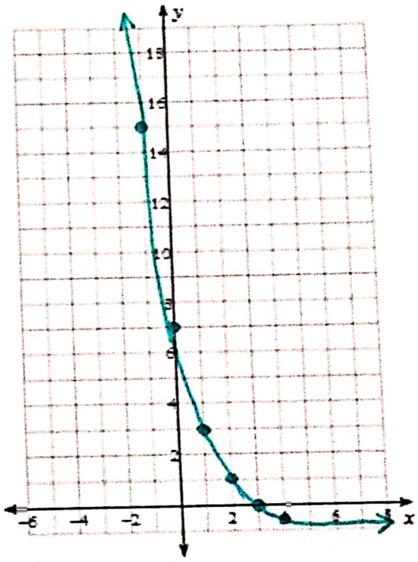
2. $y = 2\left(\frac{1}{2}\right)^x - 4$

   Vertical asymptote @ y = −4

   Domain: $(-\infty, \infty)$

   Range: $(-4, \infty)$

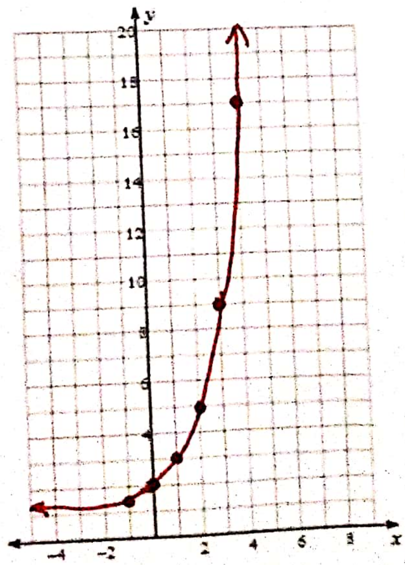
(#3) Graph $f(x) = 4\left(\frac{1}{2}\right)^{x-1} - 1$. Then use this graph to identify the following key features.



- Domain: $(-\infty, \infty)$
- Range: $(-1, \infty)$
- Increasing/Growth or Decreasing/Decay
- Asymptote at $y = -1$
- Y-intercept at 7
- End behavior:

$x \to \infty, y \to$ −1 $\quad x \to -\infty, y \to$ ∞

(#4) Graph $f(x) = 4(2)^{x-2} + 1$. Then use this graph to identify the following key features.



- Domain: $(-\infty, \infty)$
- Range: $(1, \infty)$
- Increasing/Growth or Decreasing/Decay
- Asymptote at $y = 1$
- Y-intercept at 2
- End behavior:

$x \to \infty, y \to$ ∞ $\quad x \to -\infty, y \to$ 1